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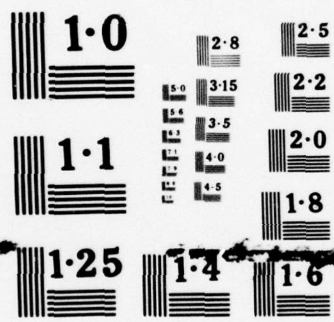

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Office of Naval Research
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NR 307-352

FINAL TECHNICAL REPORT

SCALE MODEL ULTRASONIC STUDY OF ARCTIC ICE
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Washington, D.C.

April 1979

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The report contains a collection of titles and abstracts of papers published and under consideration, relative to reflectivity from ice cover.

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The experimental and theoretical work performed under this Contract was concerned with a scale model study of various properties of Arctic ice, in particuler the reflectivity of sonic signals from the sea-water/ice-cover interface. The scaling was accomplished by considering the usual thickness of the Arctic ice cover and the wavelength of the usual sonar signal, and then changing the ice thickness to a few millimeter and the frequency of the sonic signal to a few megahertz; this scaling maintains the same value of the product (frequency times thickness) for both cases, the situation in the Arctic and the scaled version of the phenomena investigated in the laboratory.

Although reflectivity coefficients can be easily calculated from time-honored formulas, these calculations do not consider the fact that plane waves of infinite extent are not possible in nature, and that all signals do have a finite dimension, i.e., one invariably deals with a finite, bounded beam of sonic signals. This departure from the simplistic view creates new problems in the evaluation of reflectivity, and thus a great part of the work performed was concerned with the formulation of appropriate expressions describing the actual situation. This formulation was then experimentally verified for easily handled interfaces, namely nonmelting plates immersed and also floating on water. This was done initially, before experiments with ice plates were conducted, in order to check out the basic premises of the newly developed theoretical approach to the problem. The next step was to check into the angular dependence of the incident/reflected beam ratio, and it was found that here too the plane wave theory leads to useless results, and the bounded beam approach had to be investigated. This was again accomplished, first for ice-simulating plates and water interfaces and then for ice plates of millimeter thickness.

In the course of these investigations it was realized that the initial amplitude of the impinging signal enters the description of the reflection if non-

linear processes occur during the propogation in the water or upon reflection. These frequency dependent and amplitude dependent phenomena became increasingly important and non-negligible in the course of the work that these processes warranted a separate investigation by themselves. Thus, with the agreement of the Code sponsoring the present Contract, a renewal proposal was submitted to Code 421 for the expressed purpose to investigate the parameters of nonlinearity, in a most general manner, before a continuation and conclusion of the present contract could be considered in the previously anticipate manner.

Nevertheless, the present Contract has produced a number of results we had anticipated, and these results are listed in this Report. The present Contract actually consisted of two seperate activities: Contract N00014-69-A-0220-007, sponsored through Code 415, and upon reorganization within ONR, Contract N00014-75-C-0333, sponsored through Code 461. Although this reorganization and numbering of the Contracts did not imply different efforts, it caused a Final Report to be submitted at the time of the reorganization. Clearly, the work was not completed in all of its phases at that time, thus the first Final Report described the work accomplished until that time. Various Technical Reports have been submitted throughout the periods covered by both Contracts, and thus this Report will refer to prior submissions by means of citing the titles and abstracts of the various publications originating from this activity in the Contract period.

The more recent results, not covered in previous Technical Reports will be discussed in more detail. In this connection it should be noted that an extension of the present Contract (no cost) was requested and this request had been granted because the final reduction of the rather complex and involved computer results generated and the comparison of experimental results obtained in the lab and presently stored on Video Tape have been most time-consuming and have necessitated computation adjustments previously thought to be minor. Thus the present

report is a Final Report to which an Addendum will be submitted as soon as the data reduction and comparison is completed.

Section I of this Report contains copies of titles and abstracts of work performed and already submitted to the sponsoring Code. Some comments concerning continuity and/or reasons for performing the listed work are included in this section. Section II describes the work not reported on in Section I and accomplished up to the expiration of the Contract.

Section I

The preliminary findings during the first few months of the Contract were concerned with an overall investigation of the parameters which play a role in the determination of ice thickness via reflection from the Arctic ice cover. These findings were reported on in Technical Report No. 1.

SCALE MODEL ULTRASONIC STUDY OF ARCTIC ICE

TECHNICAL REPORT NO. 1

①

Backscattering of ultrasonic signals from solid plates is interpreted to yield information about the flexural modes of the plate and the sonic velocities in sea ice as well as the fractional ice content of the water/sea-ice layer.

It became apparent that the shear velocity in ice influences the reflection properties of the ice to a great extent, and since this velocity is ordinarily of no great interest in other Arctic work, little information was available about it. Therefore, as a pre-requisite for completing the work, an investigation was undertaken which resulted in the following paper:

Determination of Transverse Wave Velocity in Transition Layer of Sea Ice From Reflection of Water-Borne Sound

②

Measurement of reflection coefficients at water-sea ice interfaces were reported by *Langleben* [1970], and the values of the sound velocity in the skeleton layer as well as its density were deduced from the data. Langleben used a formula given by *Rayleigh* [1945] that is applicable when the interface is formed by two liquids. The use of this formula resulted in the finding that the density of the skeleton layer ice was comparatively high (0.996 g cm^{-3}) and the sound velocity was low (1810 m/s). It was also noted by Langleben that for large angles of sound incidence the reflection coefficients were considerably lower than the value of unity predicted by the Rayleigh formula. These data were thus omitted from the evaluation.

A reevaluation of the data is presented here that removes some of the difficulties encountered.

It was found advantageous to have available a method with which the sonic velocities in ice or any other solid can be measured with one technique, since both of these velocities are required as data in the evaluation. This method was developed and was described in:

A Simple Technique for Simultaneous Measurement of Longitudinal and Shear Wave Velocities of Solids (3)

Sound velocities in solids can be determined indirectly by measuring ultrasonic reflection or transmission at liquid-solid boundaries. The incidence of the ultrasonic beam may be normal to the surface of the bulk solid in which case only the longitudinal sound velocity can be determined from measurements of the reflection coefficient, or the sound beam may impinge from the liquid at oblique incidence in which case one can obtain the sound velocities in the solid by observing angles of maximum reflection.

In this paper we propose a simple method in which the existence of critical angles of the solid is not a requirement nor is it necessary to use two types of transducers. With this method, both the longitudinal and the shear wave velocities in the solid are measured simultaneously via Schlieren techniques using the same incident longitudinal sound beam.

In connection with this investigation it was soon found that reflectivity at the critical angles of incidence (primarily the Rayleigh angle) is markedly different from the reflection at other, noncritical angles. A study of these drastic changes upon reflection can best be done with polycrystalline solids (not restricted to ice), and this study resulted in a paper listed below:

Ultrasonic scattering from polycrystalline solids and plates (4)

Relative maxima in ultrasonic backscattering were observed at critical angles of incidence that correspond to Lamb modes on brass plates located between air and water. Critical angles were determined utilizing schlieren visualization of beam displacement. Results are compared to backscattering at the Rayleigh angle at a brass bulk solid-water interface.

These basic results and their relation to the task at hand were described in the second Technical Report:

Sonic Reflectivity from Sea-Ice/Water Interface ⑤

Technical Report No. 2.

Reflectivity curves are calculated for sea-ice/water boundaries in which the densities and sonic velocities in the two media are changed in increments. The resulting changes in the reflectivity curves are interpreted in terms of the influence of the parameters. Analysis is given indicating under which conditions reflectivity measurements can or cannot yield information about the values of sonic velocities and densities of the sea-ice and the water. Possible misleading measurements are indicated and the influence of Rayleigh waves on measurement is described.

At this point it became apparent that nonlinear effects in the water or the ice will influence the results if the frequency and/or the amplitude of the probing sonic signal are high. In order to be able to eliminate this unwanted effect it became necessary to understand the parameters which are responsible for the presence of the effect. Although the effect initially was to be excluded from the considerations its mere presence made an investigation of it mandatory before its elimination could be gueranteed.

Since the best way of studying the effect is contained in an examination of surface waves on single crystals (due to its orientation dependent values of the surface wave velocities, unfortunately absent in polycrystalline substances like large quantities of ice), we conducted further experiments on nonlinear surface wave interactions. The results were published in

Further investigation of noncollinear surface-wave interactions ⑥

Two noncollinear surface waves of 5 and 7 MHz have been excited on the surface of single-crystal copper samples in such directions that the waves could interact via the nonlinearities of the medium to produce a third surface wave at the sum frequency of 12 MHz. Using optical techniques, the three waves involved have been detected on two crystal surfaces, the (001) and (111) planes of copper, and the amplitude of each of the waves has been measured.

These results and other obtained outside the work related to the present Contract formed the basis for the renewal proposal alluded to above.

Clearly, any deep re-investigation of the nonlinear effects would have brought the efforts outside the task of the Contract, thus we concentrated on the continuation of reflectivity work, taken into account all precautions necessary to avoid nonlinear behavior of the substances involved.

It was found that the loading of the plate does change the value of the surface wave velocity, and this velocity is an important quantity in the calculation of reflectivity. Ultimately, the surface wave velocity can be determined from an observation of the reflectivity behavior, procipally, the phase of the reflection at incidence at the Rayleigh angle. The work on the phase of the reflection was described in:

Phase of ultrasonic reflection at Rayleigh angle incidence 7

The phase of an ultrasonic beam reflected from a liquid-solid interface at and near Rayleigh angle incidence is measured using a new technique. Results indicate that two waves comprise the reflected beam, i.e., a specularly reflected wave and a reradiated surface wave, which do not change phase as the angle of incidence varies, which maintain a 180° phase difference between them, and which propagate collinearly only at the Rayleigh angle.

Approximating or simulating the properties of ice as closely as possible by using plastic plates in the inital experiments on the reflectivity showed that

a) the finite thickness of the plate and the frequency of the sonic signal used results in the existence of more than one critical angle of reflection where reflectivity data behave anomalously,

b) the reflection coefficient for bulk substances is practically unrelated to reflectivity from liquid/solid/liquid layer geometries.

These points were discussed at the London ICA under the title:

EIGHTH INTERNATIONAL CONGRESS
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ULTRASONIC RERADIATION FROM RAYLEIGH AND LAMB WAVES AT LIQUID-SOLID INTERFACES

8

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INTRODUCTION

The surface wave excited by a sound beam incident on a liquid-solid bulk interface at the Rayleigh angle reradiates energy into the liquid at the Rayleigh angle. The reradiation and specular reflection produce a resultant sound field partially described by Schoch's "beam displacement" (1), and recently more completely by Bertoni and Tamir (2).

In a similar manner, Lamb waves can be excited at liquid-solid plate interfaces to produce similar "beam displacements" (3).

THEORETICAL CONSIDERATIONS AND EXPERIMENTAL RESULTS

Phase velocities V of free-plate Lamb modes depend on the product fd (frequency x plate thickness) and the longitudinal and shear velocities of the solid. The lowest few Lamb modes are shown for brass in Fig. 1, and for Plexiglas in Fig. 2.

If the plate is immersed in water, loading changes the V. New values are found by adding, to the free-plate equations, terms containing the ratio r = [density of water/density of the solid]. V can be expressed in terms of the angle of sound incidence α, from Snell's law.

Calculations for brass ($\rho = 8.6$) show that the values of V differ very little for free or for loaded plates (Fig. 1), while for solids with $\rho \approx 1$, the two sets of curves differ markedly (Figs. 2 and 3).

Schlieren techniques are used to experimentally verify the results. For a given solid and a fixed fd, those α were determined where "beam displacement" occurred, giving values of V. Results (Figs. 2 and 3) show that a free-plate approach should not be used for plastic, ice, and other low-density plates in water.

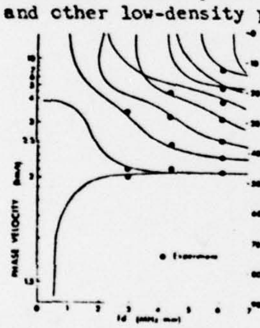

Fig.1. Modes of free and water-loaded brass plate

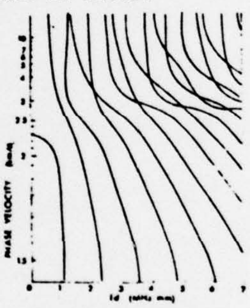

Fig.2. Free plate modes of Plexiglas

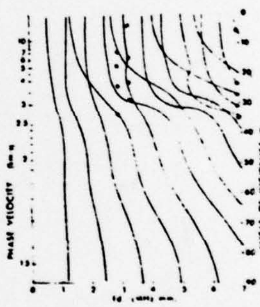

Fig.3. Modes of Plexiglas plate in water

Acknowledgment: Supported by the Office of Naval Research, U.S. Navy.

REFERENCES

(1) A. Schoch, Acustica (1952) 2 1; (2) H. Bertoni and T. Tamir, Appl. Phys. (1973) 2 157; (3) O. Diachok and W. Mayer, JASA (1973) 57 946.

Above cited publications have been included in the Final Report on Contract N00014-69-A-0220-007. In terms of the task, there was no discontinuity in effort between that Contract and the continuation Contract N00014-75-C-0333, sponsored by Code 461. The papers and results discussed below thus constitute the concluding efforts on the latter Contract, with the final results to follow in an Addendum to be submitted at the conclusion of the computation evaluation. This evaluation is now being accomplished at no additional cost to the Contract.

Section II

As was pointed out above, the determination of ice parameters, like thickness, via sonic reflectivity depends on the appropriate combination of information about the reflection properties, in general, of a bounded beam from a solid/liquid interface, and on the reflectivity parameters of ice in its various mechanical forms (i.e., density, sonic velocities). Thus at the start of the second Contract, the effort was two-fold: determine standard reflectivity for plane waves from an infinite half space combination ice/sea water, and to continue the investigation of bounded beam reflectivity in general. The first topic was investigated and the results are given in the following:

Determination of sonic velocities from reflectivity losses at sea ice/water boundaries

9

Reflectivities are calculated as functions of sonic incidence angle, densities, and sonic velocities. The resulting sets of curves are used to determine under which conditions reflectivity measurements can or cannot yield information about the values of sonic velocities and density of sea ice.

This paper describes the various differences one may expect whenever a sonic signal is reflected from the ice/sea water boundary. It does yield information about the angular ranges where the three important parameters (density, longitudinal, shear velocity) influence markedly the plane wave reflection coefficient, and where variations in any of these parameters do not significantly change the reflection coefficient magnitude.

The second topic which was investigated does relate to the problem of finite beam reflection coefficient versus plane wave reflection coefficient, and as a subdivision of these investigations, the influence of solid geometry on the finite beam width reflection. The geometry in turn can be divided into cases where

the plate reflector is bounded by the same liquid on both sides, by dissimilar liquids on the two sides of the solid plate, and by a liquid on one side and vacuum or air on the other.

The first investigation in this regard was concerned with a comparison of solid bulk versus solid plate reflectors and the types of surface waves which can be set up in the reflection process. A description of these surface waves, Rayleigh or Lamb waves is contained in

Rayleigh and Lamb waves at liquid–solid boundaries (10)

The equations governing Rayleigh and Lamb mode propagation are examined for free and for liquid-loaded solids. Examples are given to show under what conditions the free-solid approach yields acceptable solutions for the velocities and under what conditions the more involved liquid-loaded solid formulism must be used.

It was found that there are certain theoretical similarities between the surface waves on the bulk solid and on the plate solid, which were pointed out in

Theoretical similarities of Rayleigh and Lamb modes of vibration (11)

Poles of the infinite plane-wave reflection coefficient are used to show a correspondence between Rayleigh and Lamb modes of vibration. It is demonstrated that a Rayleigh vibrational mode is a special type of Lamb mode of vibration. Further, it is shown that it should be expected that one vibrational mode for a thick plate should be similar to the theoretically predicted vibrational mode of an infinite half space, a Rayleigh mode. Thus, it is consistent to use a thick plate as an approximation to an infinite half space and expect results predicted by Rayleigh-wave analysis.

Attention was then focused on nonspecular reflection, since it was found that reflection at a Lamb angle or at the Rayleigh angle is accompanied by marked changes in the profile of the reflected beam, and this nonspecular reflection in turn can be used as an indicator that a plate mode has been set up. The theory of this phenomena was given in:

Theory of Nonspecular Reflection Effects for an Ultrasonic Beam Incident on a Solid Plate in a Liquid 12

Abstract—Poles and zeroes of the infinite plane wave amplitude reflection coefficient are used to derive a theoretical prediction of the nonspecular reflection effects which have been observed for an ultrasonic beam incident on an isotropic solid plate in a liquid. It is shown that there are two types of nonspecular reflections, and they can be characterized in terms of a single parameter which requires a knowledge of the imaginary part of a pole of the infinite plane wave reflection coefficient. Theoretical predictions of nonspecular reflection intensities are presented. Finally, it is shown that the reflection characteristics for high-frequency beams incident on thick plates are the same as those expected for the reflection from a single liquid/solid interface, i.e., two *infinite* half-spaces.

This theory enables one to compare reflection from half-spaces and solid plates. I was found that non-specular reflection can then be related to plate thickness and frequency, as was shown in:

Non-specular reflection from solid plates and half-spaces 13

Recent theoretical developments for the reflection effects seen for an ultrasonic bounded beam reflected from a solid surface are presented and a discussion of the implications of these developments is related to the case of reflection from infinite half-spaces and solid plates.

The next obvious question which arose was concerned with the relation of reflection to transmission. Although this relationship does exist, a clearcut theoretical analysis is rather difficult, and thus the topic was not actively followed after publication of the following:

Ultrasonic bounded beam reflection and transmission effects at a liquid/solid-plate/liquid interface (14)

Schlieren techniques are used to show that bounded beam reflection effects occur at liquid/solid-plate/liquid (L/SP/L) interfaces which are analogous to the bounded beam reflection effects reported previously at liquid/solid (L/S) interfaces. At L/SP/L interfaces, nongeometric effects are shown to be present in both the reflected and transmitted beams when the incident angle of an ultrasonic beam corresponds to the Lamb angle. In addition, similarities between wave phenomena at L/SP/L interfaces and L/S interfaces are presented which suggest that the description of bounded beam reflection derived by Bertoni and Tamir for a L/S interface can be qualitatively applied to the L/SP/L interface case. This theory is shown to account for the lateral extent to which a leaky Lamb surface wave, excited by mode conversion, propagates.

The determination of plate thickness by means of reflection measurement at onlique angles of incidence was found to become more involved whenever the product frequency time thickness became larger than about three (in units of meter time kHz). Thus, a study was undertaken of the reflectivity and mode structure for this region, contained:

NON-SPECULAR ULTRASONIC REFLECTION FROM THICK SOLID PLATES. (15)

Non-specular reflections of a bounded ultrasonic beam from thick plates in water are observed. It is shown that the reflected beam profile does not only depend on the product frequency of the ultrasound times plate thickness, but it ultimately depends on the imaginary part of the Lamb pole location. Furthermore, results indicate that a thick plate may not be a valid approximation of an infinite half-space, as has been commonly assumed.

The nonlinear behavior of Lamb modes on plates was investigated for solid plates in order to determine whether or not a plate can indeed support a non-linear wave. This clearly was of great importance since the actual signals used

in the Arctic may contain a number of frequencies, and it was not at all clear whether the ice plate would or could act as a frequency filter. The basic study of the behavior of the plate in response to forced nonlinear oscillations was reported on in:

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NONLINEARITIES IN LAMB WAVES (16)

The equations of motion become nonlinear for finite amplitude Lamb modes propagating in isotropic plates. Hence, mode interactions may occur. Two such interactions are the Lamb mode three-phonon interaction and Lamb mode second harmonic generation. These two interactions are investigated experimentally.

This latter contribution discusses basic concepts and results, and it together with other results in connection with nonlinear phenomena formed the basis of the renewal proposal to Code 421.

The final phases of the Contract work were concerned with the ultimate goal of the contract: the study of the ice response when it is bounded by water on one side but air on the other. Again, the investigation of the magnitude of the reflection coefficient from such an asymmetric system is an indicator of the ice thickness. Thus the formulism for this system had to be developed first before the already available experimental data (scaled-down ice plates) could be evaluated. The theoretical study is included in this Report in its entirely.

Based on these findings we investigated the experimental details of thickness determination from the reflection of bounded width ultrasonic beams from different thicknesses of ice. The results of the determination of the node structure of the ice plates are now being compared to the computation model. This model (making use of a computer "Simplex" method) was used to calculate numerically the mode structure of the system air/ice plate/sea water. A great number of difficulties inherent in the computer method made the conclusion of the program impossible within the estimated time. However, some of the preliminary results addressing

the air/ice/water system have been published. Concerning the group and phase velocity of the surface wave created in the process of critical angle reflectivity, a paper was given:

> Sonic phase and group velocity in ice plates floating on water.. Peter H. Huang and Walter G. Mayer (Physics Department, Georgetown University, Washington, DC 20057)
>
> An expression for the phase velocity dispersion of vibrational modes for an ice plate floating on water is obtained which is consistent with the seemingly different expressions contained in the reflection coefficients given by various authors. Examples for different ice parameters are given, and the dispersion relations for the group velocities is shown to be more sensitive to variations in ice properties than is the phase velocity. [Work supported by the Office of Naval Research, U.S. Navy.] (17)

More recently it was found that there exist severe restrictions on the generation of the leaky Rayleigh mode when a combination of water and ice is used as the boundary. This result had not been expected and was found on theoretical grounds very recently:

> Investigation of the conditions for the existence of a leaky Rayleigh wave. W. G. Mayer, N. G. Brower, and D. E. Himberger (Department of Physics, Georgetown University, Washington, DC 20057) (18)
>
> The Rayleigh wave, an inhomogeneous surface wave, exists for all isotropic elastic solids. When the free surface of the solid is bounded by a liquid, a leaky Rayleigh wave, or inhomogeneous damped interface wave, may exist. In the limit that the density of the liquid goes to zero, the leaky Rayleigh wave tends to the free Rayleigh wave. However, the leaky Rayleigh wave does not exist for all liquid/isotropic solid systems. A well-known condition for the existence of the leaky Rayleigh wave is that the velocity of sound in the liquid must be less than that of the shear in the elastic solid. Upon investigation of the secular equation for the leaky Rayleigh wave velocity, other necessary conditions for existence become apparent. The conditions are influenced by the density ratio and the ratios of the various velocities in the liquid/solid system. [Work supported by ONR.]

The implications of this last finding are now used in the Simplex computer method to arrive at meaningful results for the thickness measurement. However, as was pointed out above, this new analysis is not complete at the present time. The final results will be submitted as an addendum to this report as soon as the work is completed.

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Plane Wave Reflection from a Plate Immersed in and Floating on a Liquid

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Summary

The reflection coefficient for a solid plate bounded by dissimilar fluids is derived. The expression can be made to be consistent with Brekhovskikh's n-layered system reflection coefficient and can be reduced to the reflection coefficient for a two-layered system derived by Schoch and to the dispersion equation for a liquid-solid-vacuum system given by Ewing, Jardetsky and Press provided one takes into account the appropriate forms of wave propagation and coordinates of the systems.

Reflexion ebener Wellen an einer beidseitig flüssigkeitsbegrenzten Platte

Zusammenfassung

Es wird der Reflexionsfaktor einer festen, durch unterschiedliche Flüssigkeiten begrenzten Platte hergeleitet. Der Ausdruck kann in Übereinstimmung mit Brekhovskikhs Reflexionsfaktor für ein n-schichtiges System gebracht und auf den von Schoch abgeleiteten Reflexionsfaktor für ein zweischichtiges System zurückgeführt werden. Berücksichtigt man die entsprechenden Formen der Wellenausbreitung und der Systemkoordinaten, so kann die angegebene Beziehung ferner auf die von Ewing, Jardetsky und Press angegebene Dispersionsgleichung für ein System Flüssigkeit-Festkörper-Vakuum zurückgeführt werden.

Réflexion d'une onde plane par une plaque immergée et flottant entre deux liquides

Sommaire

On calcule le coefficient de réflexion d'une plaque solide illimitée séparant deux fluides différents. L'expression trouvée est en concordance avec le coefficient de réflexion déduit du système à n couches de Brekhovskikh et peut être ramené également au coefficent trouvé par Schoch pour un système à deux couches ainsi qu'à l'équation de dispersion pour un système liquide-solide-vide établie par Ewing, Jardetsky et Press, pourvu qu'on donne aux propagations d'ondes et aux coordonnées des systèmes les formes convenables.

1. Introduction

In recent years a number of theoretical and experimental studies of the propagation phenomena of elastic waves in a two-layered or a three-layered system have emerged which make use of an expression for the reflection coefficient or a dispersion equation for plate mode velocities. A three-layered system is defined by a solid plate bounded by dissimilar fluids. If the solid plate is bounded on both sides by the same fluid, it is called a two-layered system. The reflection coefficient is referred to the amplitude ratio of the reflected wave with respect to the incident wave. The dispersion equation of a layered system is contained in the denominator of the reflection coefficient, and the plate mode velocities can be obtained by setting the denominator equal to zero. Schoch [1] obtained an expression for the reflection coefficient by using a particle-displacement formalism and applied it in an experimental study [2] of a two-layered system. Ewing, Jardetsky and Press [3] gave the dispersion equation for a floating solid plate without including the effect of the air bounding the plate on one side. Later Brekhovskikh [4] obtained an expression for the reflection coefficient for an n-layered system, and thus also for a three-layered system, by using a particle-potential formalism.

Unfortunately, Brekhovskikh's expression for a three-layered system does not reduce to the two-layered system given by Schoch. The differences occur in the sign of the imaginary terms and therefore the location of a pole of the reflection coefficient in the wave-vector plane. Also, Brekhovskikh's expression for the dispersion equation for a three-layered system obtained from the denominator of the reflection coefficient does not reduce to the form for a liquid-solid-vacuum system given by Ewing et al. The difference appears that the latter's expression does not contain imaginary terms explicitly while the other formulas do.

It is, therefore, the object of this article to resolve the problem of discrepancies. Extending Schoch's treatment to derive the reflection coefficient for a three-layered system Huang [5] had taken into ac-

count the spatial coordinates with respect to the selection of direction of wave propagation in the layered media. It is shown here that Brekhovskikh's expression of reflection coefficient for a three-layered system can be made to be consistent with Huang's expression and reduces to Schoch's expression for a two-layered system, and that the expression of Ewing, Jardetsky and Press for the dispersion equation for a liquid-solid-vacuum system can be made to be consistent with both the corresponding Brekhovskikh's and Huang's expression, provided one takes into account the choice of the coordinates system and appropriate forms of wave propagation.

2. Derivation

2.1. *The derivation from Brekhovskikh's expression to Huang's expression and the reduction to Schoch's expression*

Assuming a wave propagation in the form

$$\exp[+\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)], \quad \text{where} \quad \boldsymbol{x}=x\hat{x}+z\hat{z},$$

and using the spatial coordinates as shown in Fig. 1a, Schoch [1] obtained the following reflection coefficient (disregarding the phase angle $\omega d\cos\theta_1/c_1$) for a two-layered system with plate thickness d:

$$R = \frac{(X-\mathrm{i}Z_1\sin\gamma\sin\varepsilon)(Y-\mathrm{i}Z_1\cos\gamma\cos\varepsilon)+\mathrm{i}JZ_1}{(X-\mathrm{i}Z_1\sin\gamma\sin\varepsilon)(Y+\mathrm{i}Z_1\cos\gamma\cos\varepsilon)}, \tag{1}$$

where

$$X \equiv \cos^2(2\beta)\cos\gamma\sin\varepsilon + 4(c_s/c_l)^2 \times \sin\alpha\cos\alpha\sin\beta\cos\beta\sin\gamma\cos\varepsilon,$$

$$Y \equiv \cos^2(2\beta)\sin\gamma\cos\varepsilon + 4(c_s/c_l)^2 \times \sin\alpha\cos\alpha\sin\beta\cos\beta\cos\gamma\sin\varepsilon,$$

$$J \equiv \cos^2(2\beta)\sin\varepsilon\cos\varepsilon + 4(c_s/c_l)^2 \times \sin\alpha\cos\alpha\sin\beta\cos\beta\sin\gamma\cos\gamma,$$

$$Z_1 \equiv \varrho_1 c_1\cos\alpha/\varrho_p c_l\cos\theta_1$$

with

$$\gamma \equiv \omega d\cos\alpha/2c_l,$$

$$\varepsilon \equiv \omega d\cos\beta/2c_s.$$

The angles θ_1, α, β are connected by Snell's law

$$\frac{\sin\theta_1}{c_1} = \frac{\sin\alpha}{c_l} = \frac{\sin\beta}{c_s}, \tag{2}$$

where θ_1 is the incident angle in medium 1, α and β are the angles formed by the respective normals to the longitudinal and shear wave fronts with the positive z-axis, c_1 is the wave velocity in the liquid,

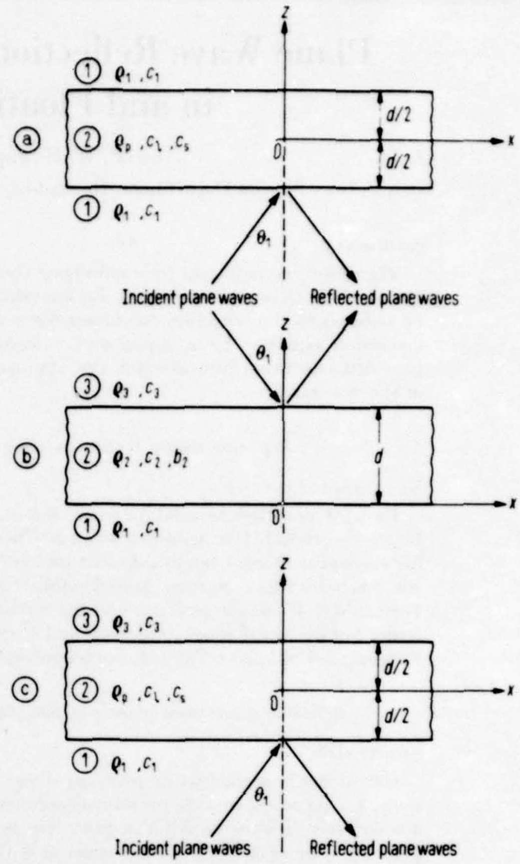

Fig. 1. Spatial coordinates with respect to the incident and the reflected plane waves used by (a) Schoch for a two-media system, (b) Brekhovskikh for a three-media system, and (c) Huang for a three-media system.

c_l and c_s are the respective longitudinal and shear wave velocities in the solid plate.

Considering the same waveform but a different coordinate system with respect to the incident and the reflected waves as shown in Fig. 1b, Brekhovskikh [4] obtained the following reflection for a three-layered system, based on his expression for an n-layered system

$$V = \frac{M(Z_1-Z_3)+\mathrm{i}[(N^2-M^2)Z_3-Z_1]}{M(Z_1+Z_3)+\mathrm{i}[(N^2-M^2)Z_3+Z_1]}, \tag{3}$$

where

$$N \equiv \frac{Z_2\cos^2(2\gamma_2)}{Z_1\sin P} + \frac{Z_{2t}\sin^2(2\gamma_2)}{Z_1\sin Q},$$

$$M \equiv \frac{Z_2\cos^2(2\gamma_2)\cot P}{Z_1} + \frac{Z_{2t}\sin^2(2\gamma_2)\cot Q}{Z_1},$$

$$Z_1 \equiv \varrho_1 c_1/\cos\theta_1, \quad Z_3 \equiv \varrho_3 c_3/\cos\theta_3$$

with

$$Z_2 \equiv \varrho_2 c_2/\cos\theta_2, \quad Z_{2t} \equiv \varrho_2 b_2/\cos\gamma_2,$$
$$P \equiv \omega d \cos\theta_2/c_2, \quad Q \equiv \omega d\cos\gamma_2/b_2.$$

The corresponding Snell's law associated with eq. (3) to eq. (2) is as follows:

$$\frac{\sin\theta_1}{c_1} = \frac{\sin\theta_2}{c_2} = \frac{\sin\gamma_2}{b_2}. \tag{4}$$

Extending Schoch's work, Huang [5] derived the following reflection coefficient for a three-layered system as shown in Fig. 1c by taking into account the positive z-direction to be either upward or downward and the wave propagation to be either of the form

$$\exp[+\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)] \quad \text{or}$$
$$\exp[-\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)],$$

where $\boldsymbol{x} = x\hat{x} + z\hat{z}$:

$$R = \frac{(2X \mp 2\mathrm{i}\bar{Z}_3 \sin\gamma\sin\varepsilon)(Y \mp \mathrm{i}\bar{Z}_1\cos\gamma\cos\varepsilon) \pm \mathrm{i}J(\bar{Z}_1 + \bar{Z}_3)}{(2X \mp 2\mathrm{i}\bar{Z}_3 \sin\gamma\sin\varepsilon)(Y \pm \mathrm{i}\bar{Z}_1\cos\gamma\cos\varepsilon) \mp \mathrm{i}J(\bar{Z}_1 - \bar{Z}_3)}, \tag{5}$$

where

$$\bar{Z}_3 \equiv \varrho_3 c_3 \cos\alpha/\varrho_p c_1 \cos\theta_3,$$

θ_3 is the angle between the normal to the transmitted wave front in medium 3 and the positive z-axis; X, Y, J, $\bar{Z}_1$, γ and ε are defined as in eq. (1). Choosing the positive z-direction to be upward, the upper sign in eq. (5) should be used if the wave propagation in the media is described by

$$\exp[+\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)],$$

and the lower sign if the wave is given by

$$\exp[-\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)].$$

Reversing the direction of the positive z-axis, the upper sign should be used in eq. (5) if the wave is given by the negative exponential, and the lower sign for the wave described by the positive exponential.

Following this rule, eq. (3) can be rewritten as shown in eq. (6) under the transformation of the physical system pictured in Fig. 1b to that in Fig. 1c,

$$V = \frac{M(Z_3 - Z_1) - \mathrm{i}[(N^2 - M^2)Z_1 - Z_3]}{M(Z_3 + Z_1) - \mathrm{i}[(N^2 - M^2)Z_1 + Z_3]}. \tag{6}$$

By multiplying both the denominator and the numerator by a common factor $2\mathrm{i}(Z_1/Z_2{}^2)$, eq. (3) can be written as

$$V = \frac{2(Z_1/Z_2{}^2)[\mathrm{i}M(Z_3 - Z_1) + (N^2 - M^2)Z_1 - Z_3]}{2(Z_1/Z_2{}^2)[\mathrm{i}M(Z_3 + Z_1) + (N^2 - M^2)Z_1 + Z_3]}. \tag{7}$$

By adding and subtracting some identical terms, the denominator of eq. (7) becomes

$$\begin{aligned}
&2[(Z_1/Z_2)^2(N^2 - M^2) + \mathrm{i}M(Z_1/Z_2{}^2)\times\\
&\times(Z_1 + Z_3) + Z_3Z_1/Z_2{}^2] =\\
&= 2[(Z_1/Z_2)^2N^2 - (Z_1/Z_2)^2M^2 -\\
&- (Z_1/Z_2)^2MN + (Z_1/Z_2)^2MN +\\
&+ \mathrm{i}(Z_1/Z_2)^2M + \mathrm{i}(Z_3Z_1/Z_2{}^2)M +\\
&+ \mathrm{i}(Z_1/Z_2)^2N - \mathrm{i}(Z_3Z_1/Z_2{}^2)N +\\
&+ Z_3Z_1/Z_2{}^2] - 2\mathrm{i}(Z_1/Z_2)^2N +\\
&+ 2\mathrm{i}(Z_3Z_1/Z_2{}^2)N =\\
&= 2(Z_1/Z_2)[N + M - \mathrm{i}(Z_3/Z_1)]\times\\
&\times(Z_1/Z_2)(N - M + \mathrm{i}) -\\
&- 2\mathrm{i}(Z_1/Z_2)N[(Z_1/Z_2) - (Z_3/Z_2)].
\end{aligned} \tag{8}$$

Similarly, the numerator of eq. (7) can be written as

$$\begin{aligned}
&2[(Z_1/Z_2)^2(N^2 - M^2) + \mathrm{i}M(Z_1/Z_2{}^2)\times\\
&\times(Z_3 - Z_1) - Z_3Z_1/Z_2{}^2] =\\
&= 2(Z_1/Z_2)[N + M - \mathrm{i}(Z_3/Z_1)]\times\\
&\times(Z_1/Z_2)(N - M - \mathrm{i}) +\\
&+ 2\mathrm{i}(Z_1/Z_2)N[(Z_1/Z_2) + (Z_3/Z_2)].
\end{aligned} \tag{9}$$

Now

$$\begin{aligned}
&(Z_1/Z_2)[N + M - \mathrm{i}(Z_3/Z_1)] =\\
&= [\cos^2(2\gamma_2)\cos(P/2)/\sin(P/2)] +\\
&+ [(b_2/c_2)(\cos\theta_2/\cos\gamma_2)\times\\
&\times\sin^2(2\gamma_2)\cos(Q/2)/\sin(Q/2)] -\\
&- \mathrm{i}\varrho_3c_3\cos\theta_2/\varrho_2c_2\cos\theta_3,
\end{aligned} \tag{10}$$

$$\begin{aligned}
&(Z_1/Z_2)(N - M \pm \mathrm{i}) =\\
&= [\cos^2(2\gamma_2)\sin(P/2)/\cos(P/2)] +\\
&+ [(b_2/c_2)(\cos\theta_2/\cos\gamma_2)\times\\
&\times\sin^2(2\gamma_2)\sin(Q/2)/\cos(Q/2)] \pm\\
&\pm \mathrm{i}\varrho_1c_1\cos\theta_2/\varrho_2c_2\cos\theta_1,
\end{aligned} \tag{11}$$

and

$$\begin{aligned}
&2\mathrm{i}(Z_1/Z_2)N =\\
&= \mathrm{i}\{[\cos^2(2\gamma_2)/\sin(P/2)\cos(P/2)] +\\
&+ (b_2/c_2)(\cos\theta_2/\cos\gamma_2)\times\\
&\times\sin^2(2\gamma_2)/\sin(Q/2)\cos(Q/2)]\}.
\end{aligned} \tag{12}$$

By making a change of the following corresponding notations used in eq. (3) and (5),

$$\gamma_2 \to \beta, \quad P/2 \to \gamma, \quad Q/2 \to \varepsilon, \quad b_2 \to c_s,$$
$$c_2 \to c_1, \quad \theta_2 \to \alpha, \quad \varrho_2 \to \varrho_p.$$

Eqs. (10), (11) and (12) become, respectively,

$$(Z_1/Z_2)[(N + M - \mathrm{i}(Z_3/Z_1)] = \\ = [\cos^2(2\beta)\cos\gamma/\sin\gamma] + 4(c_s/c_1)^2 \times \\ \times \sin\alpha\cos\alpha\sin\beta\cos\beta(\cos\varepsilon/\sin\varepsilon) - \\ - \mathrm{i}Z_3 = (X/\sin\gamma\sin\varepsilon) - \mathrm{i}Z_3, \qquad (10')$$

$$(Z_1/Z_2)(N - M \pm \mathrm{i}) = \\ = [\cos^2(2\beta)\sin\gamma/\cos\gamma] + 4(c_s/c_1)^2 \times \\ \times \sin\alpha\cos\alpha\sin\beta\cos\beta(\sin\varepsilon/\cos\varepsilon) \pm \\ \pm \mathrm{i}Z_1 = (Y/\cos\gamma\cos\varepsilon) \pm \mathrm{i}Z_1 \qquad (11')$$

and

$$2\mathrm{i}(Z_1/Z_2)N = \mathrm{i}\{[\cos^2(2\beta)/\sin\gamma\cos\gamma] + \\ + 4(c_s/c_1)^2\sin\alpha\cos\alpha\sin\beta\cos\beta/\sin\varepsilon\cos\varepsilon\} = \\ = \mathrm{i}J/\sin\gamma\cos\gamma\sin\varepsilon\cos\varepsilon, \qquad (12')$$

where Snell's law, eq. (4), has been used.

Substituting eqs. (10'), (11') and (12') into eq. (8), one obtains for the denominator of eq. (7)

vacuum system pictured in Fig. 2 where the wave propagation is of the form $\exp[+\mathrm{i}(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})]$.

$$P(2Q + \delta\cosh\nu_1 H\cosh\nu_1' H) + \\ + Q\delta\sinh\nu_1 H\sinh\nu_1' H = 0. \qquad (15)$$

Eq. (15) can be rewritten as follows:

$$2(PQ + P\delta\cosh\nu_1 H\cosh\nu_1' H) + \\ + \delta(-P\cosh\nu_1 H\cosh\nu_1' H + \\ + Q\sinh\nu_1 H\sinh\nu_1' H) = 0, \qquad (16)$$

where

$$P \equiv (\nu_1'^2 + k^2)^2\cosh\nu_1 H\sinh\nu_1' H - \\ - 4\nu_1\nu_1' k^2\sinh\nu_1 H\cosh\nu_1' H,$$

$$Q \equiv (\nu_1'^2 + k^2)^2\sinh\nu_1 H\cosh\nu_1' H - \\ - 4\nu_1\nu_1' k^2\cosh\nu_1 H\sinh\nu_1' H,$$

$$\delta \equiv \varrho_2\alpha_2^2(\nu_1'^2 - k^2)(\nu_2^2 - k^2)\nu_1/\varrho_1\beta_1^2\nu_2$$

$$2[(Z_1/Z_2)^2(N^2 - M^2) + \mathrm{i}M(Z_1/Z_2^2)(Z_1 + Z_3) + Z_3 Z_1/Z_2^2] = \\ = \frac{(2X - 2\mathrm{i}Z_3\sin\gamma\sin\varepsilon)(Y + \mathrm{i}Z_1\cos\gamma\cos\varepsilon) - \mathrm{i}J(Z_1 - Z_3)}{\sin\gamma\cos\gamma\sin\varepsilon\cos\varepsilon}. \qquad (13)$$

Substituting eqs. (10'), (11') and (12') into eq. (9), one obtains for the numerator of eq. (7)

$$2[(Z_1/Z_2)^2(N^2 - M^2) + \mathrm{i}M(Z_1/Z_2^2)(Z_3 - Z_1) - Z_3 Z_1/Z_2^2] = \\ = \frac{(2X - 2\mathrm{i}Z_3\sin\gamma\sin\varepsilon)(Y - \mathrm{i}Z_1\cos\gamma\cos\varepsilon) + \mathrm{i}J(Z_1 + Z_3)}{\sin\gamma\cos\gamma\sin\varepsilon\cos\varepsilon}. \qquad (14)$$

Finally, by dividing eq. (14) by eq. (13), one thus obtains exactly the same expression as eq. (5) in accordance with the rules concerning the sign of the imaginary terms as discussed previously.

Thus, Brekhovskikh's expression eq. (3), is consistent with Huang's expression, eq. (5), provided one takes into account the coordinates of the system with respect to the incident and the reflected waves.

Next, as a special case when the solid plate is bounded on both sides by the same liquid, i.e., a two-layered system as shown in Fig. 1a, the normalized impedances of medium 1 and medium 3 are the same, namely, $Z_3 = Z_1$. It is now evident from the derivation shown above that both eqs. (3) and (5) reduce to an expression which is identical to Schoch's expression, eq. (1).

2.2. *The reduction of Ewing', Jardetsky and Press' expression to Brekhovskikh's and Huang's expression*

Eq. (15) gives Ewing, Jardetsky and Press' expression [3] of dispersion equation for a liquid-solid-

with

$$\nu_1 \equiv (k^2 - k_{\alpha_1}^2)^{1/2}, \quad \nu_1' \equiv (k^2 - k_{\beta_1}^2)^{1/2}, \\ \nu_2 \equiv (k^2 - k_{\alpha_2}^2)^{1/2}.$$

The corresponding notations used in eqs. (16) and (5), respectively, are as follows:

$$\varrho_2 \leftrightarrow \varrho_1, \quad \varrho_1 \leftrightarrow \varrho_p, \quad \alpha_1 \leftrightarrow c_1, \\ \alpha_2 \leftrightarrow c_1, \quad \beta_1 \leftrightarrow c_s, \quad H \leftrightarrow d/2,$$

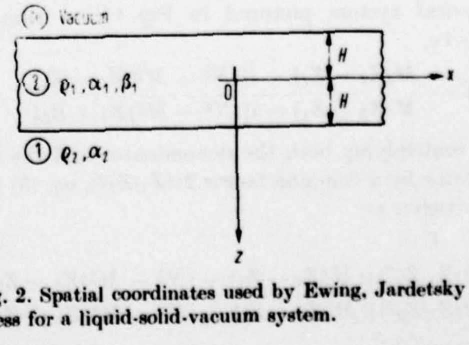

Fig. 2. Spatial coordinates used by Ewing, Jardetsky and Press for a liquid-solid-vacuum system.

$$\nu_1 \leftrightarrow -\mathrm{i}\,\omega\cos\alpha/c_1\,,\quad \nu_1' \leftrightarrow -\mathrm{i}\,\omega\cos\beta/c_s\,,$$
$$\nu_2 \leftrightarrow -\mathrm{i}\,\omega\cos\theta_1/c_1\,.$$

It is noted that Snell's law is associated with eq. (16), and its connection with angles α, β and θ_1 in eq. (5) viz.,

$$k = k_{\alpha_1}\sin\alpha = k_{\beta_1}\sin\beta = k_{\alpha_2}\sin\theta_1\,,$$

has been used to obtain the corresponding terms ν_1, ν_1' and ν_2. Hence, in eq. (16), one obtains

$$\begin{aligned}\cosh\nu_1 H &= [\exp(-\mathrm{i}\,\omega d\cos\alpha/2c_1) + \\ &\quad + \exp(+\mathrm{i}\,\omega d\cos\alpha/2c_1)]/2 = \\ &= \cos(\omega d\cos\alpha/2c_1) = \\ &= \cos\gamma\,,\end{aligned} \tag{17}$$

$$\begin{aligned}\cosh\nu_1' H &= [\exp(-\mathrm{i}\,\omega d\cos\beta/2c_s) + \\ &\quad + \exp(+\mathrm{i}\,\omega d\cos\beta/2c_s)]/2 = \\ &= \cos(\omega d\cos\beta/2c_s) = \\ &= \cos\varepsilon\,,\end{aligned} \tag{18}$$

$$\begin{aligned}\sinh\nu_1 H &= [\exp(-\mathrm{i}\,\omega d\cos\alpha/2c_1) - \\ &\quad - \exp(+\mathrm{i}\,\omega d\cos\alpha/2c_1)]/2 = \\ &= -\,\mathrm{i}\sin\gamma\,,\end{aligned} \tag{19}$$

$$\begin{aligned}\sinh\nu_1' H &= [\exp(-\mathrm{i}\,\omega d\cos\beta/2c_s) - \\ &\quad - \exp(+\mathrm{i}\,\omega d\cos\beta/2c_s)]/2 = \\ &= -\,\mathrm{i}\sin\varepsilon\,,\end{aligned} \tag{20}$$

$$(\nu_1'^2 + k^2) = (2k^2 - k_{\beta_1}^2)^2 = k_{\beta_1}^4\cos^2(2\beta)\,, \tag{21}$$

$$\begin{aligned}&-4\nu_1\nu_1' k^2 = -4(-\mathrm{i}\,\omega\cos\alpha/c_1)\times \\ &\times(-\mathrm{i}\,\omega\cos\beta/c_s)(\omega\sin\alpha/c_1)(\omega\sin\beta/c_s) = \\ &= 4k_{\beta_1}^4(c_s/c_1)^2\sin\alpha\cos\alpha\sin\beta\cos\beta\,,\end{aligned} \tag{22}$$

$$\begin{aligned}\delta &= [\varrho_1(-\mathrm{i}\,\omega\cos\alpha/c_1)/\varrho_p(-\mathrm{i}\,\omega\cos\theta_1/c_1)]\times \\ &\quad\times(c_1/c_s)^2(k_{\beta_1}^2)(\omega/c_1)^2 = \\ &= k_{\beta_1}^4\varrho_1 c_1\cos\alpha/\varrho_p c_1\cos\theta_1 = \\ &= k_{\beta_1}^4 Z_1\end{aligned} \tag{23}$$

and

$$\begin{aligned}P &= -\,\mathrm{i}k_{\beta_1}^4[\cos^2(2\beta)\cos\gamma\cos\varepsilon + \\ &\quad + 4(c_s/c_1)^2\sin\alpha\cos\alpha\sin\beta\cos\beta\times \\ &\quad\times\sin\gamma\cos\varepsilon] = \\ &= -\,\mathrm{i}k_{\beta_1}^4 X\,,\end{aligned} \tag{24}$$

$$\begin{aligned}Q &= -\,\mathrm{i}k_{\beta_1}^4[\cos^2(2\beta)\sin\gamma\cos\varepsilon + \\ &\quad + 4(c_s/c_1)^2\sin\alpha\cos\alpha\sin\beta\cos\beta\times \\ &\quad\times\cos\gamma\sin\varepsilon] = \\ &= -\,\mathrm{i}k_{\beta_1}^4 Y\,.\end{aligned} \tag{25}$$

where X and Y are as defined in eq. (5).
Now

$$\begin{aligned}&PQ = (-\mathrm{i}k_{\beta_1}^4 X)(-\mathrm{i}k_{\beta_1}^4 Y) = -k_{\beta_1}^8 XY\,,\\ &P\delta\cosh\nu_1 H\cosh\nu_1' H = \\ &= (-\mathrm{i}k_{\beta_1}^4 X)(k_{\beta_1}^4 Z_1)\cos\gamma\cos\varepsilon = \\ &= -\,\mathrm{i}k_{\beta_1}^8 X Z_1\cos\gamma\cos\varepsilon\end{aligned} \tag{27}$$

and

$$\begin{aligned}&-P\cosh\nu_1 H\cosh\nu_1' H + \\ &+Q\sinh\nu_1 H\sinh\nu_1' H = \\ &= \mathrm{i}k_{\beta_1}^4[\cos^2(2\beta)\cos\gamma\sin\varepsilon + 4(c_s/c_1)^2\times \\ &\quad\times\sin\alpha\cos\alpha\sin\beta\cos\beta\sin\gamma\cos\varepsilon]\times \\ &\quad\times\cos\gamma\cos\varepsilon + [\cos^2(2\beta)\sin\gamma\cos\varepsilon + \\ &\quad + 4(c_s/c_1)^2\sin\alpha\cos\alpha\sin\beta\cos\beta\times \\ &\quad\times\cos\gamma\sin\varepsilon]\sin\gamma\sin\varepsilon = \\ &= \mathrm{i}k_{\beta_1}^4[\cos^2(2\beta)\sin\varepsilon\cos\varepsilon(\cos^2\gamma + \sin^2\gamma) + \\ &\quad + 4(c_s/c_1)^2\sin\alpha\cos\alpha\sin\beta\cos\beta\times \\ &\quad\times\sin\gamma\cos\gamma(\cos^2\varepsilon + \sin^2\varepsilon)] = \\ &= \mathrm{i}k_{\beta_1}^4[\cos^2(2\beta)\sin\varepsilon\cos\varepsilon + 4(c_s/c_1)^2\times \\ &\quad\times\sin\alpha\cos\alpha\sin\beta\cos\beta\sin\gamma\cos\gamma] = \\ &= \mathrm{i}k_{\beta_1}^4 J\,,\end{aligned} \tag{28}$$

where J is as defined in eq. (5).

Substituting eqs. (26), (27) and (28) into eq. (16), one obtains

$$\begin{aligned}&2(-k_{\beta_1}^8 XY - \mathrm{i}k_{\beta_1}^8 X Z_1\cos\gamma\cos\varepsilon) + \\ &\quad + \mathrm{i}k_{\beta_1}^8 J Z_1 = 0\,.\end{aligned} \tag{29}$$

Since $k_{\beta_1}^8 \neq 0$, thus eq. (29) becomes

$$2X(Y + \mathrm{i}Z_1\cos\gamma\cos\varepsilon) - \mathrm{i}JZ_1 = 0\,. \tag{30}$$

Next, consider eq. (5) for the case where medium 3 is a vacuum, i.e., $Z_3 = 0$. It follows from eq. (5) that the reflection coefficient for the case where the wave is of the form $\exp[+\mathrm{i}(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})]$ and the spatial coordinates as shown in Fig. 2 can be written as:

$$R = \frac{2X(Y - \mathrm{i}Z_1\cos\gamma\cos\varepsilon) + \mathrm{i}JZ_1}{2X(Y + \mathrm{i}Z_1\cos\gamma\cos\varepsilon) - \mathrm{i}JZ_1}\,. \tag{31}$$

The dispersion equation for the liquid-solid-vacuum system is therefore given by the vanishing of the denominator of eq. (31), leading to exactly the same expression as eq. (30).

It has been shown that Ewing, Jardetsky and Press' expression, eq. (16), is indeed consistent with eq. (31), provided one takes the coordinates of the system and the forms of wave propagation into account.

Because Brekhovskikh's expression, eq. (3), is consistent with Huang's expression, eq. (5), therefore Ewing, Jardetsky, and Press' expression, eq. (16), is also in agreement with Brekhovskikh's expression for a liquid-solid-vacuum system.

3. Conclusion

In this article, we have reviewed various expressions of reflection coefficient and dispersion equation given by Schoch [1], Brekhovskikh [4], Ewing et al. [3] and Huang [5]. The result is established that it is possible to make these expressions consistent with one another provided one takes into account the coordinates of the physical systems and the appropriate forms of wave propagation.

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References

[1] Schoch, A., Acustica **2** [1952], 1.
[2] Schoch, A., Acustica **2** [1952], 17.
[3] Ewing, W. M., Jardetsky, W. S. and Press, F., Elastic waves in layered media. McGraw-Hill, New York 1957, Ch. 6.
[4] Brekhovskikh, L. M., Waves in layered media. Academic Press, New York 1960, Ch. 6.
[5] Huang, P. H., Ultrasonic reflection from a three-layered system. Ph. D. Thesis, Georgetown University 1977.

Bibliography

The following papers were published under sponsorship of the present Contract. The numbers refer to the listings cited in this Report.

1 W. G. Mayer, Technical Report no. 1 (1975).
2 W. G. Mayer, J. Geophys. Res. 79, 885 (1974).
3 M. Behravesh, T. J. Plona, W. G. Mayer, Acustica 31, 241 (1974).
4 O. I. Diachok, W. G. Mayer, J. Acoust. Soc. Am. 53, 946 (1973).
5 W. G. Mayer, Technical Report No. 2 (1975).
6 W. J. Tanski, W. G. Mayer, J. Appl. Phys. 44, 506 (1973).
7 T. J. Plona, W. G. Mayer, Appl. Phys. Lett. 23, 536 (1973).
8 W. G. Mayer, T. J. Plona, M. Behravesh, 8 ICA 2, 406 (1974).
9 W. G. Mayer, M. Behravesh, T. J. Plona, JASA 57, 39 (1975).
10 T. J. Plona, M. Behravesh, W. G. Mayer, Ultras. 13,171 (1975).
11 L. E. Pitts, T. J. Plona, W. G. Mayer, JASA 60, 374 (1976).
12 L. E. Pitts, T. J. Plona, W. G. Mayer, IEEE-SU Trans. 24, 101 (77).
13 L. E. Pitts, W. G. Mayer, Ultras. 15, 201 (1977).
14 T. J. Plona, L. E. Pitts, W. G. Mayer, JASA 59, 1324 (1976).
15 M. Behravesh, L. E. Pitts, W. G. Mayer, Acoust. Lett. 1, 165 (77).
16 N. G. Brower, W. G. Mayer, 9 ICA 2, 747 (1977).
17 P. H. Huang, W. G. Mayer, JASA 62, S83 (1977).
18 W. G. Mayer, N. G. Brower, D. E. Himberger, JASA 64, S143 (1978).
19 P. H. Huang, W. G. Mayer, Acustica 40, 223 (1978).